

Vectors and Linear Equations - Answers

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1 Recall:

Sum of two vectors: $[a, b] + [c, d] = [a + c, b + d]$

Scalar multiplication of a vector and scalar: $a[b, c] = [ab, ac]$

2 Systems of Equations

Solve the following systems of equations for x and y .

1. $x + y = 8$
 $x = 3y$

$$(x, y) = (6, 2)$$

2. $3x + 4y = 0$
 $x - 2y = 10$

$$(x, y) = (-4, 3)$$

3. $\frac{2}{3}x - \frac{1}{9}y = 4$
 $\frac{x}{y} = 2$

$$(x, y) = \left(\frac{72}{11}, \frac{36}{11}\right)$$

4. $y = x + 1$
 $x^2 + y^2 = 5$

$$(x, y) = (1, 2), (-2, -1)$$

3 Homogeneous Linear Equations

Describe all rational solutions using vectors to the following homogeneous linear equations. For your final answer in the form $q[x, y]$, reduce x and y to lowest terms or convert them both to integers if possible. The order of the negatives does not matter.

1. $x + 2y = 0$ $\boxed{[x, y] = q[-2, 1]}$ or $\boxed{q[2, -1]}$

2. $5x - 10y + 4 = 4$ $\boxed{[x, y] = q[2, 1]}$

3. $\frac{1}{3}x + \frac{1}{12}y = 0$ $\boxed{[x, y] = q[-1, 4]}$ or $\boxed{q[1, -4]}$

4. $\sqrt{2}x + 7y - \sqrt{3} = -\sqrt{3}$ $\boxed{[x, y] = q[-7, \sqrt{2}]}$ or $\boxed{q[7, -\sqrt{2}]}$

4 Word Problems

1. What is the sum of the two vectors $[3, 8]$ and $[75, 200]$?

$$[3, 8] + [75, 200] = [3 + 75, 8 + 200] = \boxed{[78, 208]}$$

2. What is the simplified scalar multiplication of the vector $\sqrt{2}[14, 28\sqrt{2}]$?

$$\sqrt{2}[14, 28\sqrt{2}] = [14 * \sqrt{2}, 28\sqrt{2} * \sqrt{2}] = \boxed{[14\sqrt{2}, 56]}$$

3. All vectors $[x, y] = q[51, -68]$ are solutions to the homogeneous equation $ax + by = 0$, where a and b are in lowest terms. Find the value of $3a + 2b$.

$$(a, b) = (4, 3)$$

$$3a + 2b = 3(4) + 2(3) = 12 + 6 = \boxed{18}$$