Sets

Answers

November 2, 2019

Problem 1 Set Membership

Determine whether each statement is true or false. Remember that $\varnothing$ is the empty set, which has no members.

a. $1 \in \{1, 2, 3\}$ $\boxed{\text{True}}$ False

b. $0 \in \varnothing$ True $\boxed{\text{False}}$

c. $\{1, 2\} \in \{\{2, 1\}, \{3, 4\}\}$ $\boxed{\text{True}}$ False

d. $\{\varnothing\} \in \{\varnothing\}$ True $\boxed{\text{False}}$

e. $0.5 \in \left\{\frac{1}{2}\right\}$ $\boxed{\text{True}}$ False

f. $1 \in \left\{\frac{1}{2}, \frac{3}{2}\right\}$ True $\boxed{\text{False}}$

Problem 2 Set Equality

Determine whether each statement is true or false. Remember that two sets are equal if they contain exactly the same members, regardless of order. Also remember that there is no concept of multiple membership in a set: $\{0, 0\}$ is the same set as $\{0\}$.

a. $\varnothing = \varnothing$ $\boxed{\text{True}}$ False

b. $\varnothing = \{\varnothing\}$ True $\boxed{\text{False}}$

c. $\{1, 2\} = \{2, 1\}$ $\boxed{\text{True}}$ False

d. $\{1, 2, 3\} = \{3, 1, 2\}$ $\boxed{\text{True}}$ False

e. $\{1, 2\} = \{1, 2, 3\}$ True $\boxed{\text{False}}$

f. $\{1, 1\} = \{1\}$ $\boxed{\text{True}}$ False

Problem 3 Subset

Determine whether each statement is true or false. Remember that $A \subseteq B$ if all members of A are also members of B. $A \subsetneq B$ if $A \subseteq B$ but $A \neq B$, i.e. all members of A are also members of B but at least one member of B is not a member of A.

a. $\varnothing \subseteq \{1, 2\}$ $\boxed{\text{True}}$ False

b. $\{1, 2\} \subseteq \{1, 2\}$ $\boxed{\text{True}}$ False

c. $\{1, 2\} \subseteq \varnothing$ True $\boxed{\text{False}}$

d. $\{1, 2\} \subseteq \{\{1, 2\}\}$ True $\boxed{\text{False}}$

e. $\{1, 2\} \subsetneq \{1, 2, 3\}$ $\boxed{\text{True}}$ False

f. $\{1, 2\} \subsetneq \{2, 3\}$ True $\boxed{\text{False}}$

Problem 4 Set Unions and Intersections

List all the members of the following sets using set notation. The first two are done for you.

a. $\{0,1\} \cup \{1,2\} = \{0,1,2\}$

b. $\{0,1\} \cap \{1,2\} = \{1\}$

c. $\{0,1,2\} \cup \{2,1,0\} = \{\boxed{0,1,2}\}$

d. $\{0,1,2,3,4\} \cap \{0,1,4,9,16\} = \{\boxed{0,1,4}\}$

Problem 5 True

Write a brief explanation for why each of the following statements is true.

1. For all sets A and B, $A \subseteq A \cup B$. $\boxed{\text{By definition, every member of } A \text{ is in the union}}$
2. For all sets A and B, $A \cap B \subseteq A$. $\boxed{\text{By definition, every member of } A \cap B \text{ is in } A}$
3. For all sets A, B, and C, if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.
   $\boxed{\text{If all } A\text{s are } B\text{s and all } B\text{s are } C\text{s, then all } A\text{s are } C\text{s.}}$
4. For all sets A and B, $A \cap B \subseteq A \cup B$. $\boxed{\text{Combine parts a, b, and c}}$

Problem 6 False

Find a counterexample for each of the false statements below.

1. For all sets A and B, either $A \subseteq B$ or $B \subseteq A$ (or both). $\boxed{A = \{1\}}$, $\boxed{B = \{2\}}$
2. For all sets A, $\varnothing \subsetneq A$. $\boxed{A = \varnothing}$

Problem 7 Infinite Sets

Recall that:

- $\mathbf{N} = \{0, 1, 2, \ldots\}$ is the set of natural numbers.
- $\mathbf{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ is the set of integers.
- $\mathbf{Q}$ is the set of rational numbers (fractions).
- $\mathbf{R}$ is the set of real numbers.

We saw in class that $\mathbf{N} \subseteq \mathbf{Z} \subseteq \mathbf{Q} \subseteq \mathbf{R}$. Demonstrate that each of these inclusions is proper, i.e. $\mathbf{N} \subsetneq \mathbf{Z} \subsetneq \mathbf{Q} \subsetneq \mathbf{R}$.

Solution. For $\mathbf{N} \subsetneq \mathbf{Z}$, an example is -1. For $\mathbf{Z} \subsetneq \mathbf{Q}$, an example is $\frac{1}{2}$. For $\mathbf{Q} \subsetneq \mathbf{R}$, an example is $\sqrt{2}$.