# Real Numbers

Answers

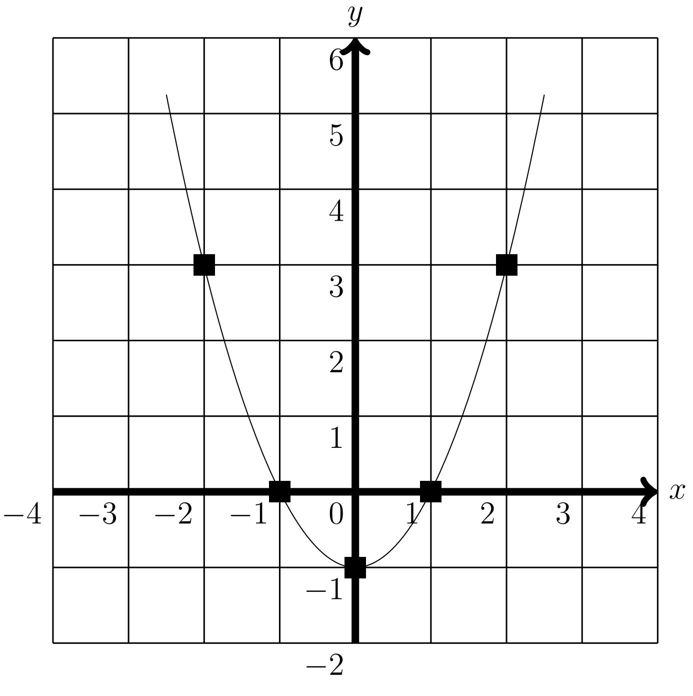
October 25, 2019

## Problem 1 Graphing a Function

In this problem, we will draw a graph for the function $y = (x-1)(x+1)$. First, fill out this table of values. The first few entries are filled out for you.

| $x$ | $y = (x-1)(x+1)$ | Rough work |
|---|---|---|
| $-2$ | $3$ | $(-2-1) \times (-2+1) = -3 \times (-1) = 3$ |
| $-1$ | $0$ | $(-1-1) \times (-1+1) = -1 \times 0 = 0$ |
| $0$ | $\boxed{-1}$ | |
| $1$ | $\boxed{0}$ | |
| $2$ | $\boxed{3}$ | |

On the grid paper below, axes have been drawn for you. Draw the five points above onto the grid. Again, the first two points are already done for you.



Now try to connect the five points using a smooth curve. Extrapolate (guess how the curve will continue) past $-2$ and $2$. You can calculate the correct values of $y$ for $x = -3$ or $x = 3$ if this helps you guess how the curve will continue. Answer the following questions using your graph:

1. What does the minimum value of $y$ look like? $\boxed{-1}$
2. If $x$ becomes very large (say, $x > 10$), what happens to $y$? $\boxed{\text{It becomes very large!}}$
3. If $x$ becomes very small (say, $x < -10$), what happens to $y$? $\boxed{\text{It becomes very large!}}$
4. If $y = 3$, what are the possible values of $x$? $\boxed{-2}$, $\boxed{2}$
5. If $y = 4$, is it possible for $x$ to be an integer? Do you think $x$ could be a rational number? Why or why not? No, it is not possible for $x$ to be an integer. $x$ cannot be a rational number either because if $4 =$

## Problem 2 Linear Equations with a Unique Solution

Using division, determine the unique possible value of $x$. Simplify your answers if possible.

a. $\sqrt{3}x = 3$ $x = \boxed{\sqrt{3}}$

b. $\sqrt{2}x = \sqrt{8}$ $x = \boxed{2}$

c. $\frac{1}{2}x = \frac{1}{\sqrt{2}}$ $x = \boxed{\sqrt{2}}$

d. $\sqrt{12}x = -2\sqrt{3}$ $x = \boxed{-1}$

## Problem 3 Exponentiation

Simplify as much as possible. Classify the answer as an integer, rational number, and/or real number. (Remember that integers are still rational numbers, and rational numbers ar still real numbers, so be sure to circle all the correct answers.)

1. $\left(\sqrt{2^3}\right)^2 = \boxed{8}$ [✓ Integer] [✓ Rational] [✓ Real]
2. $\sqrt[3]{3^9} = \boxed{27}$ [✓ Integer] [✓ Rational] [✓ Real]
3. $\sqrt{\frac{2}{98}} = \boxed{\frac{1}{7}}$ Integer [✓ Rational] [✓ Real]
4. $\sqrt{\frac{1+\sqrt{2}}{\sqrt{2}-1} - \sqrt{8}} = \boxed{\sqrt{3}}$ Integer Rational [✓ Real]

## Problem 4 Real Problems

In class, we saw that some real numbers can be obtained through using rational exponents (radicals), and adding, subtracting, and multiplying these numbers.

In 1824, Niels Henrick Abel proved that these numbers are not even enough to describe all real solutions to polynomials. An example of such a polynomial is $x^5 - x + 1 = 0$. We say that this polynomial does not have a solution in radicals.

Show that the polynomial $x^5 - x + 30 = 0$, by contrast, does have a solution in radicals. This is in fact the unique real solution for this polynomial. The solution is $-2$. This is an integer, and so is a solution in radicals