Rational Numbers

Answers

September 28, 2019

Problem 1 Equivalent Fractions

Determine, using cross-multiplication or otherwise, whether the fractions are equivalent.

a. $\frac{2}{4} = \frac{1}{2}$ $\boxed{\text{True}}$ False

b. $\frac{3}{-12} = \frac{-1}{4}$ $\boxed{\text{True}}$ False

c. $\frac{8}{7} = \frac{9}{8}$ True $\boxed{\text{False}}$

d. $\frac{-1}{-2} = \frac{-5}{10}$ True $\boxed{\text{False}}$

Problem 2 Simplify Fractions

Find the equivalent fraction with the smallest (positive) denominator. In other words, simplify the fraction. If the denominator is 1, you may leave it as a fraction, even though it is equal to an integer.

a. $\frac{2}{4} = \frac{1}{2}$

b. $\frac{3}{12} = \frac{1}{4}$

c. $\frac{10}{-15} = \frac{-2}{3}$

d. $\frac{-3}{-3} = \frac{1}{1}$

Problem 3 Fraction Operations

Compute the result of each expression and write it as a fraction with the smallest possible positive denominator. If the denominator is 1, you may leave it as a fraction, even though it is equal to an integer.

a. $\frac{1}{2} + \frac{1}{2} = \frac{1}{1}$

b. $\frac{1}{2} + \frac{-1}{3} = \frac{1}{6}$

c. $\frac{1}{-2} + \frac{-3}{-4} = \frac{1}{4}$

d. $\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$

e. $\frac{3}{8} \times \frac{5}{9} = \frac{5}{24}$

f. $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$

Problem 4 True or False

Recall that rational numbers include all fractions of integers, including those with denominator 1, so all integers are rational numbers.

1. The sum of two rational numbers is always a rational number. $\boxed{\text{True}}$ False
2. The product of two rational numbers is always a rational number. $\boxed{\text{True}}$ False
3. There are multiple ways to write every rational number as a fraction. $\boxed{\text{True}}$ False
4. For all integers a, b, c, d, if $b \neq 0 \neq d$,

$$\frac{a}{b} + \frac{c}{d} = \frac{ac}{bd}$$

True $\boxed{\text{False}}$

Problem 5 Pizza Confusion

Melek and Zahari each have a pizza, but they are not the same size. $\frac{2}{5}$ of Melek's pizza has the same mass as $\frac{3}{8}$ of Zahari's pizza. If Zahari's pizza is 320 g, then what is the mass of Melek's pizza? $\boxed{300\,\text{g}}$

Problem 6 Integral Root Theorem

The **integral root theorem** says that if $x : \mathbf{Z}$ is an integer satisfying a **polynomial equation** constraint

$$a_0 + a_1 x + \cdots + a_{n-1} x^{n-1} + x^n = 0$$

where n is a positive integer and each $a_0, a_1, \ldots, a_{n-1}$ is an integer, then x is a (positive or negative) factor of a_0. For example, if $3 - 4x + x^2 = 0$, the only possible solutions for x are the factors of 3: -3, -1, 1, and 3. We can check that 1 and 3 are solutions for x that satisfy the constraint, while -1 and -3 do not work.

Using the integral root theorem, determine the integer solutions x to the polynomial equation:

$$-5 - x + 5x^2 + x^3 = 0$$

Answer: $x \in \{\boxed{-5}, \boxed{-1}, \boxed{1}\}$.

Problem 7 A Telescoping Sum

Find the sum:

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \cdots + \frac{1}{99 \times 100} = \frac{99}{100}$$