

Quadratic Equations

Answers

November 16, 2019

Problem 1 Quadratic Equations with a Unique Solution

Determine the unique possible real value of x . Why do these equations have only one real solution?

a. $x^2 - 2x + 1 = 0$ (Hint: $x^2 - 2x + 1 = (x - 1)^2$) $x = \boxed{1}$

b. $x^2 + \frac{2}{3}x + \frac{1}{9} = 0$ (Hint: $x^2 + \frac{2}{3}x + \frac{1}{9} = (x + \frac{1}{3})^2$) $x = \boxed{-\frac{1}{3}}$

Problem 2 Quadratic Equations with No Real Solution

Explain why the following quadratic equations do not have any real solutions.

a. $x^2 = -1$ (Hint: Is the square of a real number ever negative?) **Solution:** The square of a real number is never negative. Therefore this equation has no solution.

b. $x^2 - \frac{5}{4}x + \frac{367}{448} = 0$ (Hint: $x^2 - \frac{5}{4}x + \frac{367}{448} = (x - \frac{5}{8})^2 + \frac{3}{7}$) **Solution:** The hint tells us that this equation, if it had a real solution x , would mean that $(x - \frac{5}{8})^2 = -\frac{3}{7}$ is negative, but that is not possible!

Problem 3 A Tale of Two Cities

Phitsanulok and Sukhothai are two small cities in Thailand. Bangkok is the capital of Thailand. Assume, for simplicity, that the Earth is flat for this question. It's not, of course! But when we are at relatively small scales, this assumption is not so terrible.

1. Phitsanulok is 400 km north of Bangkok. Draw this information on the map below, where Bangkok is labelled for you already. We will say that Bangkok is at $(0, 0)$ and Phitsanulok is at $(0, 400)$.
2. Sukhothai is 430 km away from Bangkok, but we do not know the direction. Draw a circle around Bangkok representing the possible locations of Sukhothai, based on this information.
3. The equation of a circle centred at $(0, 0)$ is $x^2 + y^2 = r^2$, where r is the radius. If Sukhothai is at (x, y) , write down an equation that is satisfied by Sukhothai: $x^2 + y^2 = \boxed{176400}$. (Hint: $430^2 = 184900$)
4. Sukhothai is 150 km away from Phitsanulok, but we do not know the direction. Draw a circle around Phitsanulok representing the possible locations of Sukhothai, based on this information.

5. The equation of a circle centred at $(0, 400)$ is $x^2 + (y - 400)^2 = r^2$, where r is the radius. Write down another equation that is satisfied by Sukhothai: $x^2 + (y - 400)^2 = \boxed{22500}$. (Hint: $150^2 = 22500$)
6. We have narrowed down the possible locations of Sukhothai! At how many points do the circles intersect?
7. Do you think this is related to the fact that the equations we wrote are quadratic? False
8. To solve this system of equations, we can subtract the first equation from the second. This will make the x^2 term disappear (why?). We are left with $y^2 - (y - 400)^2 = 800y - 160000 = 162400$. This is actually a linear equation for y , so it should have only one solution. That is, wherever Sukhothai is, we know exactly how far north or south it is, even if we do not know for sure how far to the east or west it is. Does this match your picture? False
9. We can then substitute the value for y from the above linear equation, which is $y = 403$, into one of the quadratic equations. It does not matter which one (why?). We will get, after some simplification, the quadratic equation $x^2 = 22491$. How many real solutions x are there? How can we solve this equation? (The solutions are not integers.) **Solution:** There are two real solutions. We can solve this equation by taking negative and positive square roots.

