Linear Equations

Answers

October 5, 2019

Problem 1 Linear Equations with a Unique Solution

Using division, determine the unique possible value of x. Express any fractions in simplest form.

a. $2x = 6$ $x = \boxed{3}$

b. $-3x = 12$ $x = \boxed{-4}$

c. $\frac{1}{2}x = \frac{1}{4}$ $x = \boxed{\frac{1}{2}}$

d. $5x = 3$ $x = \boxed{\frac{3}{5}}$

Problem 2 All Roads Lead To Rome

Khulan, Alessa, and Clara are on vacation in Italy. Read Khulan's journal. For each question, decide whether we have enough information to answer it. If we do, use a linear equation or otherwise to solve the problem.

Rome is the capital of Italy. I arrived in Venice, Italy and took the train to Rome. Alessa and Clara arrived in Naples, Italy and also took the train to Rome. The train from Venice takes 4 *times as long as the train from Naples. Clara told me she spent* 40€ *on her train ticket. If you add up the amount of time that each of us spent on the train, we spent a total of* 6 *hours combined, which is not bad considering the distance! Naples is* 185 km *away from Rome. For comparison, Paris and Rome are* 1100 km*) apart. While in Rome, we tried a lot of coffee. Alessa spent twice as much on coffee as me, and Clara spent twice as much on coffee as Alessa! We spent a total of* 14€ *on coffee.*

1. How much time does the Naples–Rome train take? Yes. $t + t + 4t = 6\,\text{h} \Rightarrow t = 1\,\text{h}$
2. How much time does the Venice–Rome train take? (Did we have to write a new linear equation to solve this problem?) Yes. We can just use $4t = 4\,\text{h}$.How much time does the Paris–Rome train take? No. While we know the ratio of distances, we cannot assume that trains travel at the same speed.
3. How much did Alessa spend on coffee? Yes. $c + 2c + 4c = 14 \Rightarrow c = 2$, so Alessa spent 2€ on coffee.
4. How much does a cup of coffee cost in Italy? No. We do not know how many units of coffee any of them consumed.
5. How much money did Alessa spend on her train ticket? No. While we know the ratio of times, we cannot assume that train ticket cost is only based on time.

Problem 3 Number of Solutions to a Linear Equation

Suppose that a and b are known rational numbers. Which of the below statements about the number of rational solutions to $ax = b$ are true?

1. If $a = 0$, then $ax = b$ always has no rational solutions. True [False]
2. If $a \neq 0$, then $ax = b$ always has a unique rational solution. [True] False
3. If $b = 0$, then $ax = b$ always has multiple rational solutions. True [False]
4. If $a = b = 0$, then $ax = b$ always has multiple rational solutions. [True] False
5. If $a \neq 0$ but $b = 0$, then $ax = b$ always has no rational solutions. True [False]
6. If $a = 0$ but $b \neq 0$, then $ax = b$ always has no rational solutions. [True] False

Problem 4 Extra Known Variables

Suppose that a, b, and c are known rational numbers, with $a \neq b$. Find an expression that gives x in terms of these known rational numbers, if $ax = bx + c$. Is the solution unique?

Problem 5 A Homogeneous Linear System

Up until now, we have been focusing on a single unknown value, x. Consider the following system of equations (that is, all the equations are true):

$$\begin{aligned} x + y + z &= 0 \\ x + 2y + 3z &= 0 \end{aligned}$$

where x, y, z are unknown rational numbers.
Answers may vary for this problem.

1. Find one solution. $x_1 = \boxed{0}$, $y_1 = \boxed{0}$, $z_1 = \boxed{0}$
2. Find a different solution. $x_2 = \boxed{-1}$, $y_2 = \boxed{-1}$, $z_2 = \boxed{1}$
3. Take your solutions and add them together. That is, let $x := x_1 + x_2$, $y := y_1 + y_2$, $z := z_1 + z_2$. Is this again a solution? Why or why not? It is also a solution. This is because we can add the equations for x_1, y_1, and z_1 together with the equations for x_2, y_2 and z_2. Since the right hand sides are 0, this results in the same system!